On Gelfand graded commutative ring

Mohamed AQALMOUN

École Normale Supérieure- Fès

Samedi 18 Mars, 2023

First International Conference on Algebraic Topology and its Applications in Robotics, FS Meknès.

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Generalization in the graded setting.

Establish some topological and algebraic characterizations of these rings, one of which is the algebraic analogue of the Urysohn's lemma.

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The functor $G\mathrm{Spec}$

$G\mathrm{Spec} : \{G-\text{graded commutative rings}\} \to \mathrm{Top}, \ R \mapsto G\mathrm{Spec}R \ ; \ f \mapsto f^{-1}$

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If R is a G-graded Gelfand ring, then $G\mathrm{Max}(R)$ is Hausdorff.

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2. *$G\mathrm{Spec}(R)$ is a normal space.*

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2. *If $a \in R_e$, then there exists elements $b, c \in R_e$ such that*

$$(1 - ba)(1 - ca') = 0$$

where $a' = 1 - a$.

Théorème

Let R be a G-graded ring. The following statements are equivalent.

1. *R is a Gelfand graded ring.*
2. *Any two disjoint closed subsets of $G\mathrm{Spec}(R)$ can be separated by a regular function.*

Théorème

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Corollaire

Let R be a G-graded commutative ring. Then R is a Gelfand graded ring if and only if R_e is a Gelfand ring.


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