

ThmOn fixe R un anneau principal.Soit $f: (A, d_A) \rightarrow (B, d_B)$ un morphisme d'algon suppose i) $A^0 = B^0 = R, H^1(A) = H^1(B) = 0$ ii) $H^1(A, d), H^1(B, d)$ finiment engendrésiii) $H^2(B, d_B)$ R -libre.iv) A R -libre.v) $H^2(f): H^2(A, d_A) \rightarrow H^2(B, d_B)$ injective A lui existe

$$(A, d_A) \rightarrow (B, d_B)$$

$$\swarrow \quad \searrow \quad \uparrow \quad \downarrow$$

$$R\text{-module libre} \quad (A \amalg T(V), d)$$
Preuve: On va procéder par récurrence: $\forall k \geq 0$ a) $V = V^{\leq k}$ R -libre.b) $H^{\leq k}(f)$ isomorphismec) On peut prolonger f en $\psi: A \amalg T(V^{\leq k+1}) \rightarrow B$

$$\text{sq. } \psi|_{A \amalg T(V^{\leq k})} = f, H^{k+1}(\psi) \text{ surjective}$$
a) $k=0: V^0 = R$ Rappel: $C \amalg D = R \oplus C_+ \oplus C_+ \oplus D_+ \oplus (D^+ \oplus C^+) \oplus \dots$

$$A \amalg T(V^0) = A \amalg T(R) = A \amalg (R \oplus R \oplus R \oplus \dots)$$

$$= R \oplus A_+ = A$$

b) $\psi: A \amalg T V_0 = A \rightarrow B$ $H^0(\psi)$ isomorphisme $\psi = f$ $H^0(\psi): H^0(A) \rightarrow H^0(B)$ c) $H^1(\psi): H^1(A) \rightarrow H^1(B)$ surjective
$$\dots \quad H^1(A) \rightarrow H^1(B) \quad \dots$$

4b) $\pi(u) \dots$

k=1: a) $V^1 = \mathcal{O}_U(1) : H^1(U)$ isomorphisme

c) $\psi?$

$$\psi : A \rtimes T(V^0 \oplus V^{2,1}) \rightarrow B \quad \psi|_{A \rtimes T V^0} = \varphi$$

$$H^2(\psi) \text{ surjective} \quad H^2(\psi) : H^2(A \rtimes T V^0) \rightarrow H^2(B) \cong H^2(A) \rightarrow H^2(B) \xrightarrow{\text{libre}} 0$$

$$H^2(B) \cong H^2(A) \oplus R[z_1] \oplus \dots \oplus R[z_n] \oplus \text{partie torsion}$$

on considère $v_1, \dots, v_n$ $[\psi(v_i)] = [z_i]$ $|v_i|=2$ $dv_i = 0$ $d\psi(v_i) = \psi(dv_i) = 0$

$$V^{2,1} = Rv_1 \oplus \dots \oplus Rv_n \quad \psi(v_i) = z_i \Rightarrow d\psi(v_i) = \psi(dv_i) = 0$$

$$\psi : (A \rtimes T(V^0 \oplus V^1 \oplus V^{2,1}), d) \rightarrow (B, d_B) \quad R\text{-morph}$$

$$H^2(\psi) \text{ surjective} \quad d'alg$$

k=2: soit $[z] \in \ker(H^2(\psi))$

$$A \rtimes T(R \oplus V^{2,1}) \rightarrow B$$

$$z = a + w \quad a \in A, w \in V^2 \quad |w|=2$$

$$0 = dz = da + dw \quad w = \sum t_i v_i \quad dw = 0$$

$$|a|=2 \Rightarrow da = dw = 0$$

$$0 = \psi(z) = \psi(a) + \psi(w) \Rightarrow \psi(a) = -\psi(w) \Rightarrow \psi(a) = 0$$

$H^2(\psi)$ injective

$\Rightarrow H^2(\psi)$ isom

$$[\psi(a)] = -[\psi(w)]$$

$$\uparrow \quad \uparrow$$

$$\text{Im } H^2(\psi) \quad R[z_1] \oplus \dots \oplus R[z_n]$$

k=2 : a) $V^2 = V^{2,1}$

b) $H^2(\psi)$ isom

c) $\theta? \theta : A \rtimes T(V^{\leq 2} \oplus V^{3,1}) \rightarrow B$ θ extension

$$H^2(\theta) \text{ surjective de } \psi$$

$$H^3(\mathcal{U}) : H^3(A \amalg T(V^{\leq 2}), d) \rightarrow H^3(B, d_B)$$

$$H^3(B, d) = R[z_1] \oplus \dots \oplus R[z_n] \oplus \bigoplus_{n_m} R[z'_1] \oplus \dots \oplus \bigoplus_{n'_m} R[z'_m]$$

$$n_m / n_{m-1} / \dots / n_1$$

$$n'_1, \dots, n'_n, n'_1, \dots, n'_m$$

$$|v_i| = 3 : \theta(v_i) = z_i, \theta(v'_i) = z'_i$$

$$0 = dz_i = d\theta(v_i) = \theta(dv_i)$$

$$\Rightarrow \psi(dv_i) = 0$$

$$dv_i \in A \amalg TV^{\leq 2}$$

$$|dv_i| = 4$$

$$|dv_i| = 4 \quad dv_i \in A \amalg TV^{\leq 2} = A \amalg (T(R + V^2))$$

ψ

$$dv_i = a_1 + a_2 v_2 + w_2 b_2 + v'_2 w'_2$$

$$(v'_1)$$

$$\psi(dv_i) = 0$$

$$H^3(\emptyset)$$

$$\Leftrightarrow f(a_1) + f(a_2)z_2 + z_2 f(b_2) + z_4 z_5 = 0$$

On pose $dv_i = dv'_i = 0 \rightarrow \theta : A \amalg T(V^{\leq 2} \oplus V^{3'}) \rightarrow B$

Hypothèse On impose $\forall k \geq 2$ $\psi : (A \amalg T^{\leq k}, d) \rightarrow (B, d_B)$ donnée

by $a_k : V^{\leq k}$ R -libre $b_k : H^k(\mathcal{U})$ isomorphe

$c_k : \exists \psi : A \amalg T(V^{\leq k} \oplus V^{k+1}) \rightarrow B$ by

ψ prolonge ψ et $H^{k+1}(\psi)$ surjective

Etape $k+1$: prolonger ψ en θ pour $H^{k+1}(\emptyset)$ injective.

soit Z un sous-module de $(A \amalg TV^{\leq k}) / \cap \ker(d)$

$$Z \cap \ker(d) = R[z_1, \dots, z_n] : H^{k+1}(\psi)([z_i]) = 0 \text{ by } [z_i] \neq 0$$

$$\frac{Z \cap \ker(d)}{Z \cap \cap \ker(d)} = R / \bigoplus_{n_m} R \oplus \dots \oplus R / \bigoplus_{n_1} R \oplus \text{Libre}$$

$$m \leq n$$

$$\left| \frac{Z \cap K_n(d)}{Z \cap I_m(d)} \right| = \left(\frac{1}{n_m R} \right)^{1/n_{m-1} R} \dots \frac{1}{n_1 R} \quad m \leq n$$

$$Z \cap Z_m(d) = \langle x_1, \dots, x_m \rangle \quad m \leq n$$

$$\bar{x}_1 = \dots = \bar{x}_m = 0 \Leftrightarrow \begin{cases} x_i \in n_i R \\ x_i = n_i z_i \quad 1 \leq i \leq m \end{cases}$$

Induction $U = R \langle u_1, \dots, u_n \rangle \quad |u_i| = k \quad du_i = z_i$

$$x_i = n_i z_i \in I_m(d) \Rightarrow \exists y_i \text{ s.t. } dy_i = x_i$$

$$x_i = dy_i = n_i z_i = n_i du_i = d(n_i u_i)$$

$$\Rightarrow d(y_i - n_i u_i) = 0$$

On introduit $W = R w_1 \oplus \dots \oplus R w_m \quad |w_i| = k-1$
 $dw_i = y_i - n_i u_i \quad |u_i| = k, |y_i| = k$

On définit ψ in $\theta : (A \amalg T(V^{\leq k} \oplus V^{k+1} \oplus U \oplus W | d) \rightarrow (B | d_B))$

$$du_i = z_i \quad d\theta(u_i) = \theta(du_i) = \theta(z_i) = \psi(z_i)$$

$$dw_i = y_i - n_i u_i \quad d\theta(w_i) = \theta(dw_i) = \theta(y_i) - n_i \theta(u_i)$$

on montre :

- (i) $H^l(\psi) = H^l(\varphi) \quad l < k-1$
- (ii) $H^k(\psi) = H^k(\varphi)$
- (iii) $H^{k+1}(\psi)$ injective
- (iv) $\exists S : \text{et } \theta$

$$\theta : A \amalg T(V^{\leq k} \oplus V^{k+1} \oplus U \oplus W \oplus S)$$

$$H^{k+2}(\theta) \text{ surjective}$$

B

