Topological Data Analysis

Generalities and some Applications

Jaraf Mustapha

Université Internationale de Rabat

M.A.A.T

04-Mars-2017

1 Motivation

2 Introduction

3 Basic Concepts

4 Applications

5 Research Focus

6 Bibliography

Sommaire

1 Motivation

2 Introduction

3 Basic Concepts

4 Applications

5 Research Focus

6 Bibliography

Visual Perception

- **Visual perception** is the ability to interpret the surrounding environment using light in the visible spectrum reflected by the objects in the environment.

Visual Perception

- **Visual perception** is the ability to interpret the surrounding environment using light in the visible spectrum reflected by the objects in the environment.
- We believe that the various actions leading to a developed perception are carried out in five stages:

Visual Perception

- **Visual perception** is the ability to interpret the surrounding environment using light in the visible spectrum reflected by the objects in the environment.
- We believe that the various actions leading to a developed perception are carried out in five stages:

Visualization

Visual Perception

- **Visual perception** is the ability to interpret the surrounding environment using light in the visible spectrum reflected by the objects in the environment.
- We believe that the various actions leading to a developed perception are carried out in five stages:

Visualization

Structuring

Visual Perception

- **Visual perception** is the ability to interpret the surrounding environment using light in the visible spectrum reflected by the objects in the environment.
- We believe that the various actions leading to a developed perception are carried out in five stages:

Visualization

Structuring

Transfiguration

Visual Perception

- **Visual perception** is the ability to interpret the surrounding environment using light in the visible spectrum reflected by the objects in the environment.
- We believe that the various actions leading to a developed perception are carried out in five stages:

Visualization

Structuring

Transfiguration

Determination

Visual Perception

- **Visual perception** is the ability to interpret the surrounding environment using light in the visible spectrum reflected by the objects in the environment.
- We believe that the various actions leading to a developed perception are carried out in five stages:

Visualization

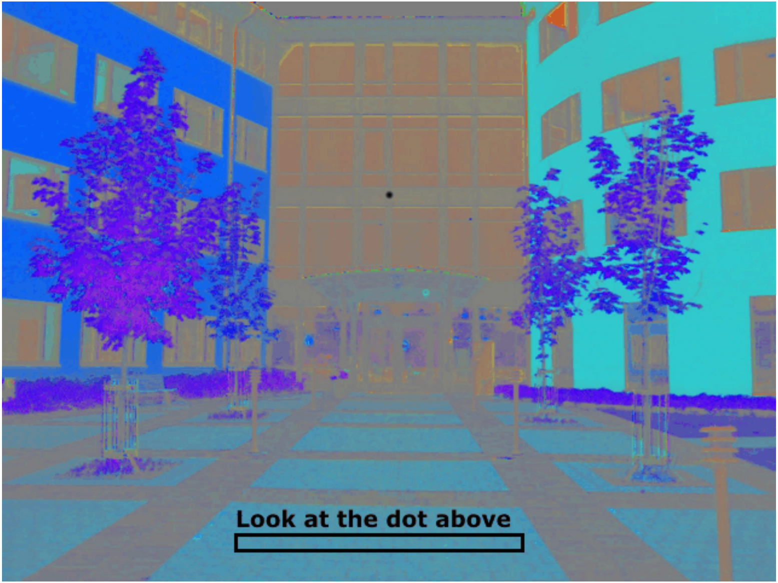
Structuring

Transfiguration

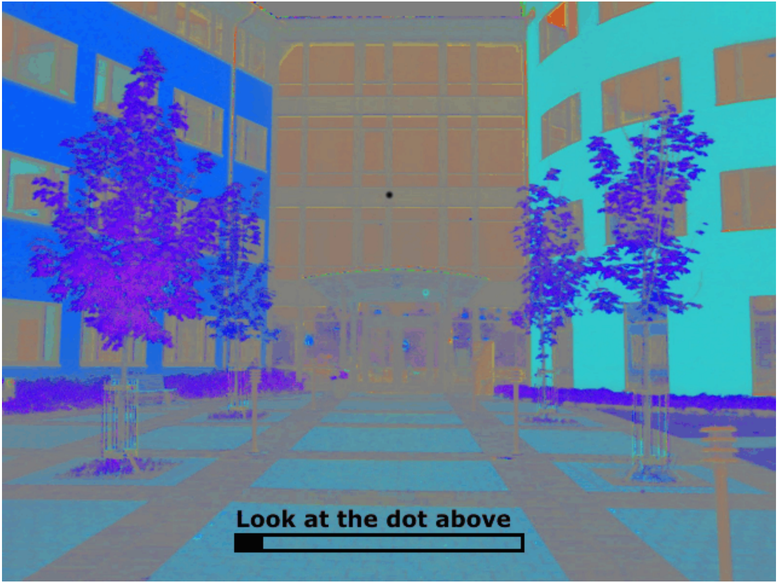
Determination

And Classification

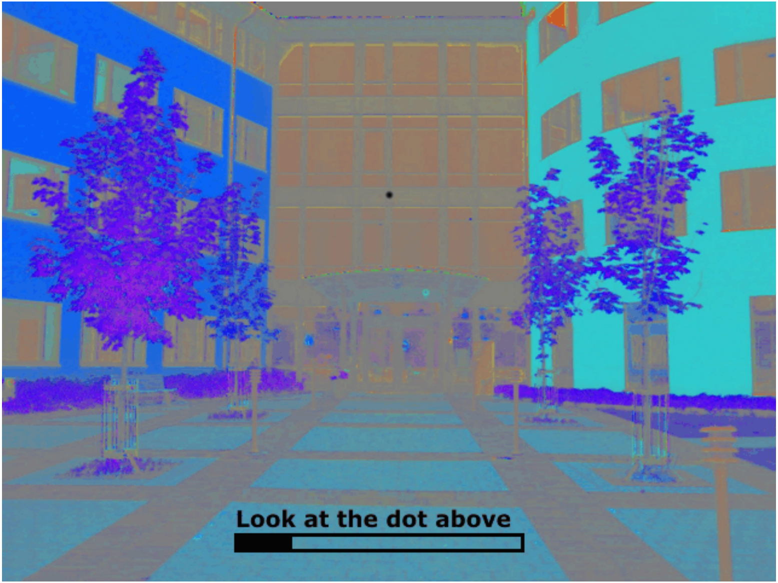
Persistence of vision



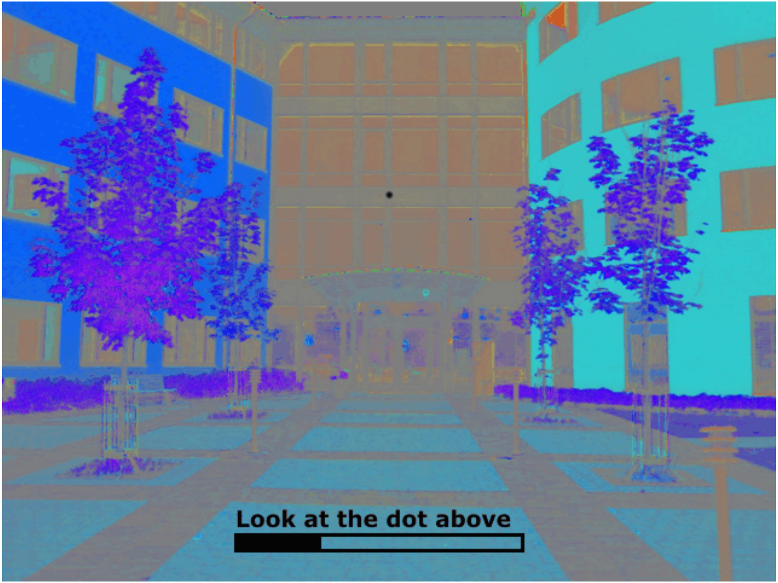
Persistence of vision



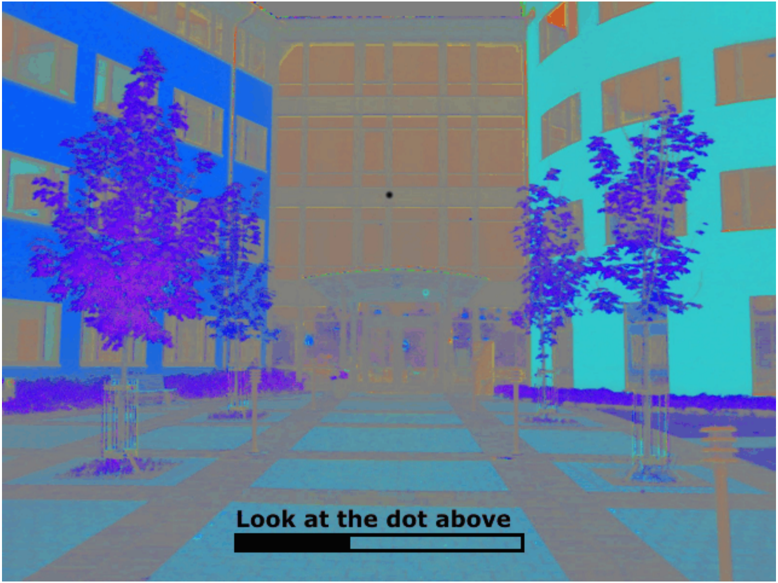
Persistence of vision



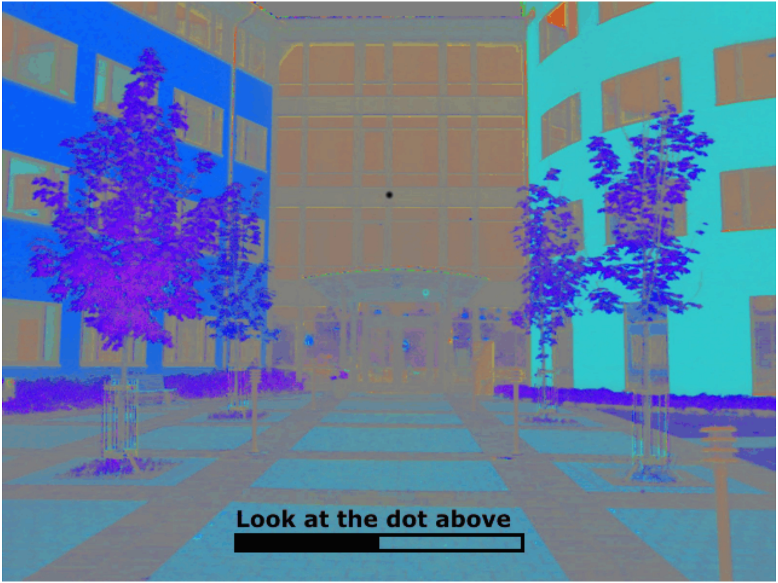
Persistence of vision



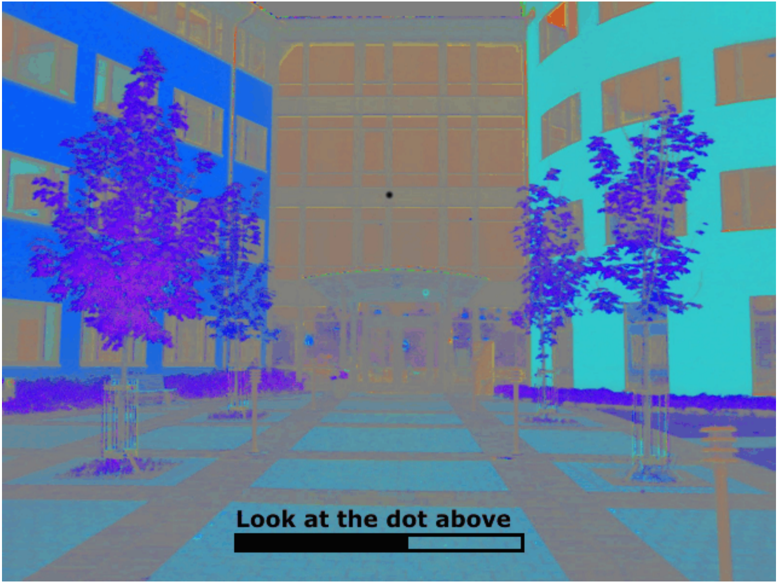
Persistence of vision



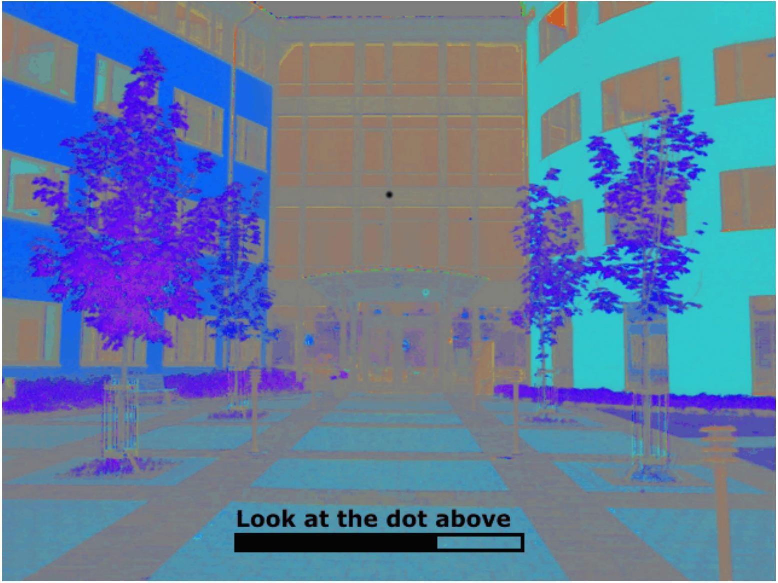
Persistence of vision



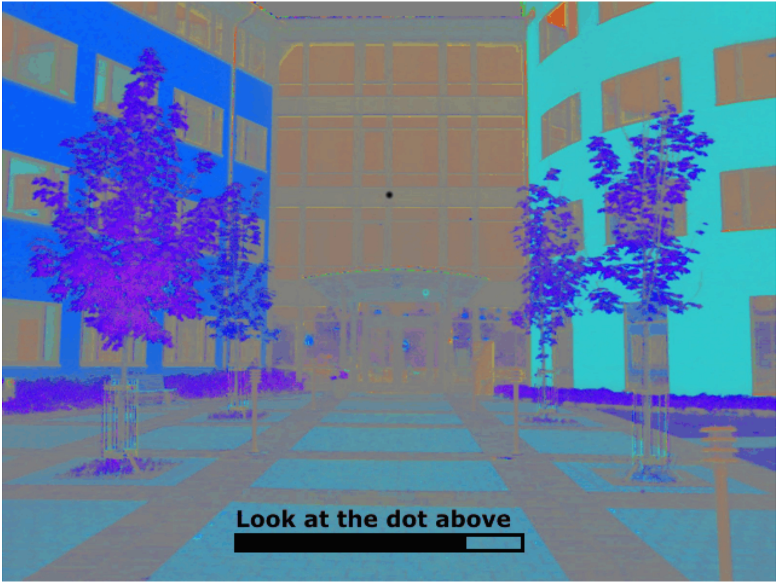
Persistence of vision



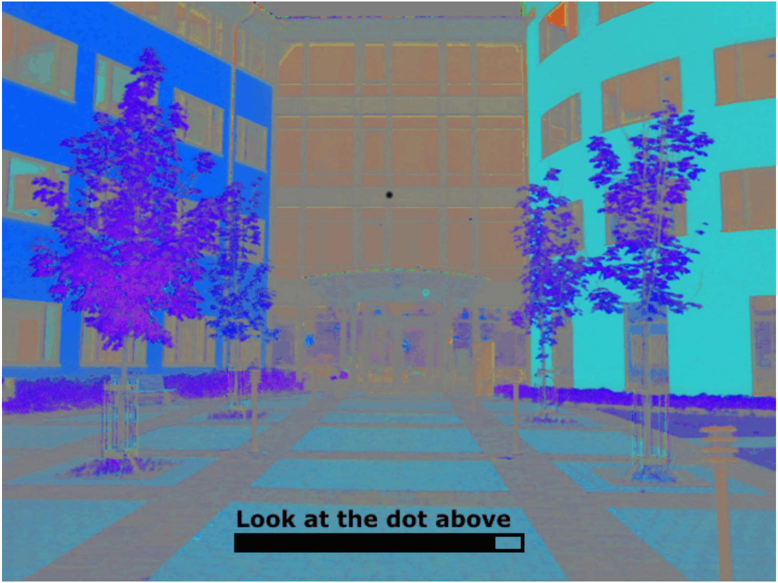
Persistence of vision



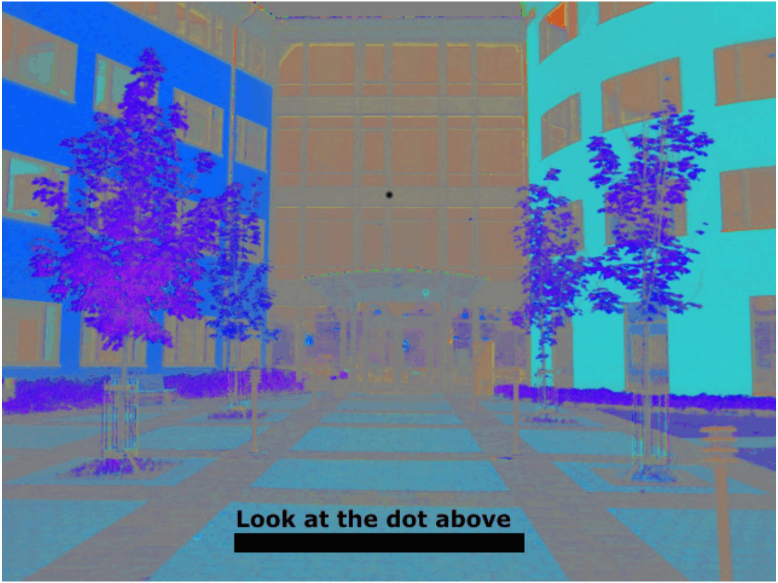
Persistence of vision



Persistence of vision

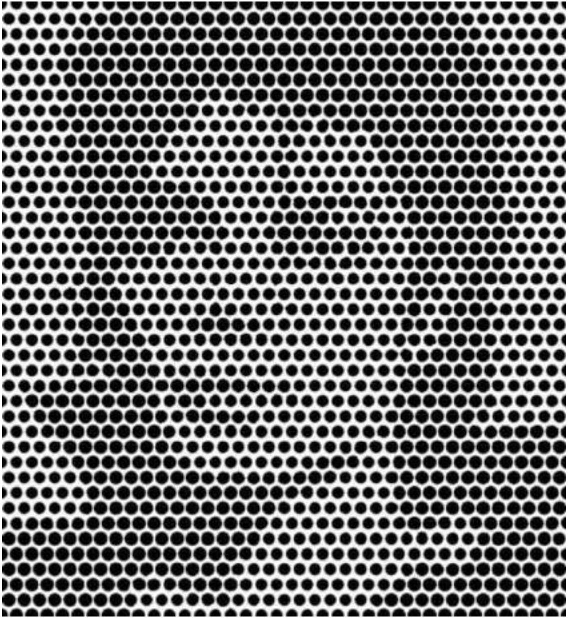


Persistence of vision



Persistence of vision

What these points looks like ?



"Data has Shape and Shape has Meaning"

G. Carlson

Sommaire

1 Motivation

2 Introduction

3 Basic Concepts

4 Applications

5 Research Focus

6 Bibliography

Topological Data Analysis

- Topological Data Analysis (TDA) is an approach to the analysis of datasets using techniques from topology.

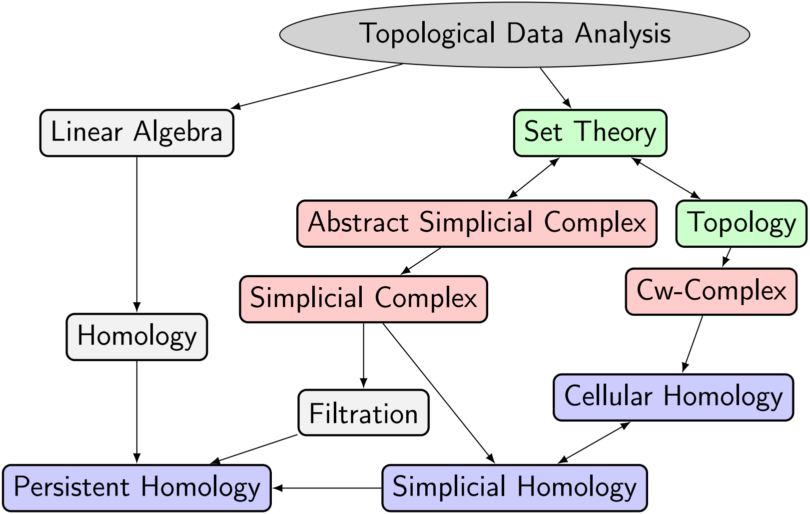
Topological Data Analysis

- Topological Data Analysis (TDA) is an approach to the analysis of datasets using techniques from topology.
- The initial motivation is to study the shape of data. TDA has combined algebraic topology and other tools from pure mathematics to give mathematically rigorous and quantitative study of "shape". The main tool is persistent homology, an adaptation of homology to point cloud data.

Topological Data Analysis

- Topological Data Analysis (TDA) is an approach to the analysis of datasets using techniques from topology.
- The initial motivation is to study the shape of data. TDA has combined algebraic topology and other tools from pure mathematics to give mathematically rigorous and quantitative study of "shape". The main tool is persistent homology, an adaptation of homology to point cloud data.
- Persistent homology has been applied to many types of data across many fields. Moreover, its mathematical foundation is also of theoretical importance. The unique features of TDA make it a promising bridge between topology and geometry.

Mathematical Dependency Tree



Sommaire

1 Motivation

2 Introduction

3 Basic Concepts

4 Applications

5 Research Focus

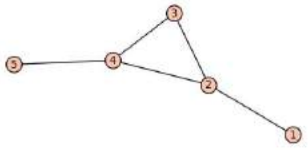
6 Bibliography

- Graph

  A (finite, combinatorial) graph is a pair (V, E), where V is a finite set and E is any collection of 2-element subsets of V.

- Graph

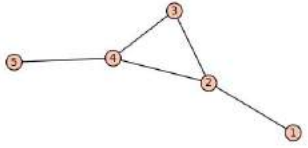
A (finite, combinatorial) graph is a pair (V, E), where V is a finite set and E is any collection of 2-element subsets of V.



- $V = \{1, 2, 3, 4, 5\}$
  $E = \{(1,2), (2,3), (3,4), (2,4), (4,5)\}$

- Graph

  A (finite, combinatorial) graph is a pair (V, E), where V is a finite set and E is any collection of 2-element subsets of V.



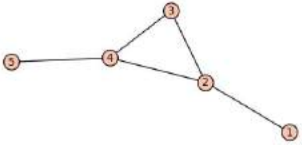
  - $V = \{1, 2, 3, 4, 5\}$
    $E = \{(1,2),(2,3),(3,4),(2,4),(4,5)\}$

- Simplicial Complex

  A (finite, combinatorial) simplicial complex is a pair (V, X) where V is a finite set and X is any collection of subsets of V such that : $Y \in X \quad and \quad Y' \subseteq Y \quad \Rightarrow \quad Y' \in X$

- Graph

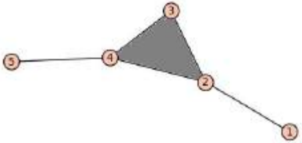
  A (finite, combinatorial) graph is a pair (V, E), where V is a finite set and E is any collection of 2-element subsets of V.



  - $V = \{1, 2, 3, 4, 5\}$
    $E = \{(1,2), (2,3), (3,4), (2,4), (4,5)\}$

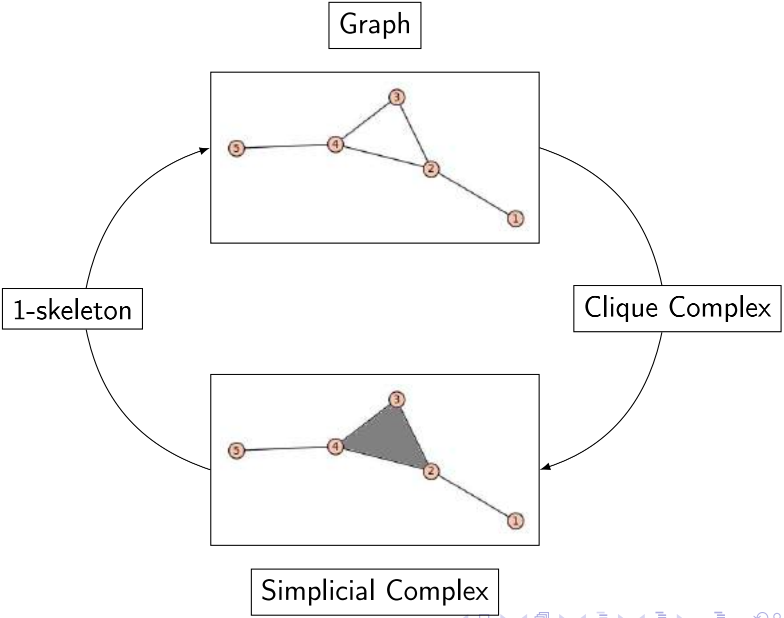
- Simplicial Complex

  A (finite, combinatorial) simplicial complex is a pair (V, X) where V is a finite set and X is any collection of subsets of V such that : $Y \in X \quad and \quad Y^{'} \subseteq Y \quad \Rightarrow \quad Y^{'} \in X$



  - $V = \{1, 2, 3, 4, 5\}$
    $E = \{(1,2), (2,3), (3,4), (2,4), (4,5), (2,3,4)\}$

Relationship

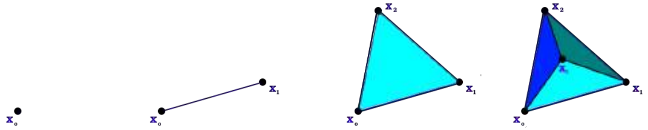


Simplicial Complexes

n-simplex

- A $(k+1)$-tuple of points in $\mathbb{R}^n$, $(x_0,...,x_n)$, where $x_i \in \mathbb{R}^n$, is said to be **affinely independent** if the set of vectors $\{\overrightarrow{x_j x_0} | 1 \leq j \leq k\}$ are linearly independent.
- An n-**simplex** is an ordered (n+1)-tuple of affinely independent point $\sigma = < x_0, ..., x_n >$.

0-Simplex | 1-Simplex | 2-Simplex | 3-Simplex

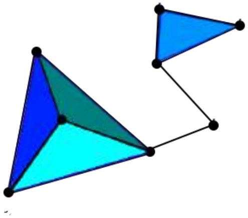


Simplicial Complex

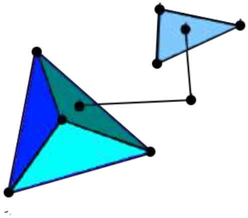
A **simplicial complex K** is a finite set of simplicies such that :

- $\sigma \in K, \quad \tau \leq \sigma \quad \Rightarrow \quad \tau \in K$
- $\sigma, \sigma' \in K \quad \Rightarrow \sigma \cap \sigma' \leq \sigma; \sigma'$

Example



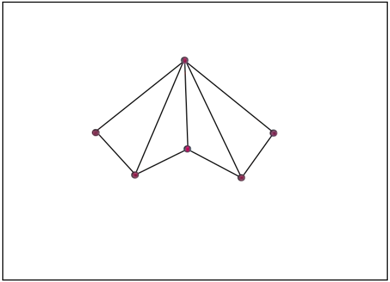
Non-Example



Betti-Numbers and Graphs

The first Betti-Number of a graph G = (V,E) with n vertices, m edges and k connected components is : $\beta_1 = m - n + k$

Figure: Connected Graph



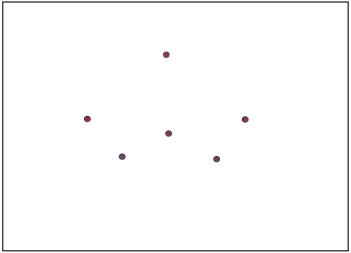
Connected Components of $\mathbb{R}^2 \setminus D$

If G = (V,E) is a combinatorial graph, $T \subseteq E$ is a spanning tree of G, and $D \subseteq \mathbb{R}^2$ is any realisation of an embedding of G in the plane, then the number of connected components of $\mathbb{R}^2 \setminus D$ is $|E| - |T|$

Betti-Numbers and Graphs

The first Betti-Number of a graph G = (V,E) with n vertices, m edges and k connected components is : $\beta_1 = m - n + k$

Figure: Set of Vertices



$k = \beta_0 = 6$

$\beta_1 = 0$

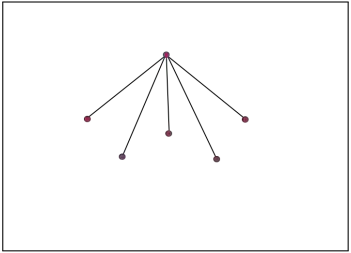
Connected Components of $\mathbb{R}^2 \setminus D$

If G = (V,E) is a combinatorial graph, $T \subseteq E$ is a spanning tree of G, and $D \subseteq \mathbb{R}^2$ is any realisation of an embedding of G in the plane, then the number of connected components of $\mathbb{R}^2 \setminus D$ is $|E| - |T|$

Betti-Numbers and Graphs

The first Betti-Number of a graph G = (V,E) with n vertices, m edges and k connected components is : $\beta_1 = m - n + k$

Figure: Spanning Tree



$\beta_0 = 1$

$\beta_1 = 0$

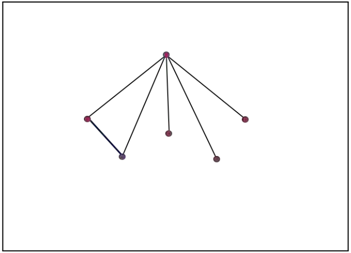
Connected Components of $\mathbb{R}^2 \setminus D$

If G = (V,E) is a combinatorial graph, $T \subseteq E$ is a spanning tree of G, and $D \subseteq \mathbb{R}^2 \setminus D$ is any realisation of an embedding of G in the plane, then the number of connected components of $\mathbb{R}^2 \setminus D$ is $|E| - |T|$

Betti-Numbers and Graphs

The first Betti-Number of a graph G = (V,E) with n vertices, m edges and k connected components is : $\beta_1 = m - n + k$

Figure: First Component



$\beta_0 = 1$

$\beta_1 = 1$

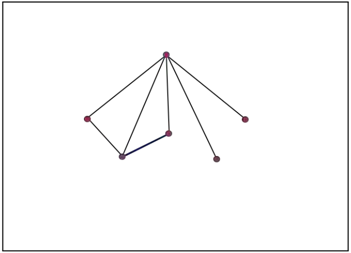
Connected Components of $\mathbb{R}^2 \setminus D$

If G = (V,E) is a combinatorial graph, $T \subseteq E$ is a spanning tree of G, and $D \subseteq \mathbb{R}^2$ is any realisation of an embedding of G in the plane, then the number of connected components of $\mathbb{R}^2 \setminus D$ is $|E| - |T|$

Betti-Numbers and Graphs

The first Betti-Number of a graph G = (V,E) with n vertices, m edges and k connected components is : $\beta_1 = m - n + k$

Figure: Second Component



$\beta_0 = 1$

$\beta_1 = 2$

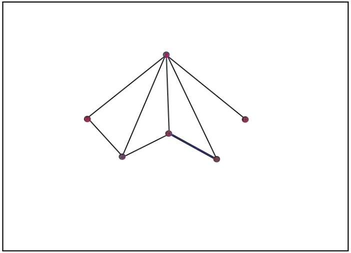
Connected Components of $\mathbb{R}^2 \setminus D$

If G = (V,E) is a combinatorial graph, $T \subseteq E$ is a spanning tree of G, and $D \subseteq \mathbb{R}^2$ is any realisation of an embedding of G in the plane, then the number of connected components of $\mathbb{R}^2 \setminus D$ is $|E| - |T|$

Betti-Numbers and Graphs

The first Betti-Number of a graph G = (V,E) with n vertices, m edges and k connected components is : $\beta_1 = m - n + k$

Figure: Third Component



$\beta_0 = 1$

$\beta_1 = 3$

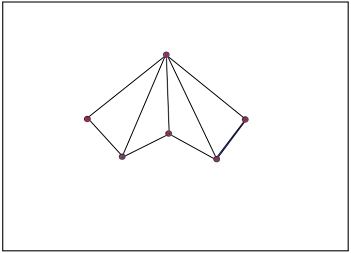
Connected Components of $\mathbb{R}^2 \setminus D$

If G = (V,E) is a combinatorial graph, $T \subseteq E$ is a spanning tree of G, and $D \subseteq \mathbb{R}^2$ is any realisation of an embedding of G in the plane, then the number of connected components of $\mathbb{R}^2 \setminus D$ is $|E| - |T|$

Betti-Numbers and Graphs

The first Betti-Number of a graph G = (V,E) with n vertices, m edges and k connected components is : $\beta_1 = m - n + k$

Figure: Last Component



$\beta_0 = 1$

$\beta_1 = 4$

Connected Components of $\mathbb{R}^2 \setminus D$

If G = (V,E) is a combinatorial graph, $T \subseteq E$ is a spanning tree of G, and $D \subseteq \mathbb{R}^2$ is any realisation of an embedding of G in the plane, then the number of connected components of $\mathbb{R}^2 \setminus D$ is $|E| - |T|$

Homology

- Homology Theory generalizes the notion of connectivity in Graph Theory to higher dimensions.

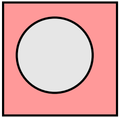
Homology

- Homology Theory generalizes the notion of connectivity in Graph Theory to higher dimensions.
- It's defined by a family of groups that capture the number of :

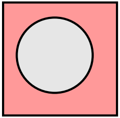
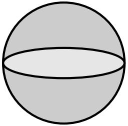
Homology

- Homology Theory generalizes the notion of connectivity in Graph Theory to higher dimensions.
- It's defined by a family of groups that capture the number of :
  - Connected components 

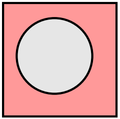
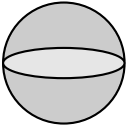
Homology

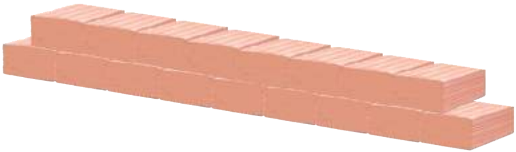
- Homology Theory generalizes the notion of connectivity in Graph Theory to higher dimensions.
- It's defined by a family of groups that capture the number of :
  - Connected components 
  - The number of holes 

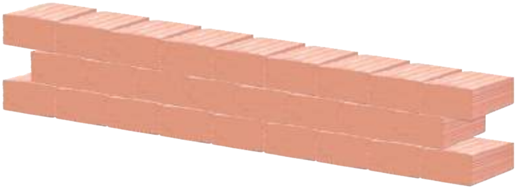
Homology

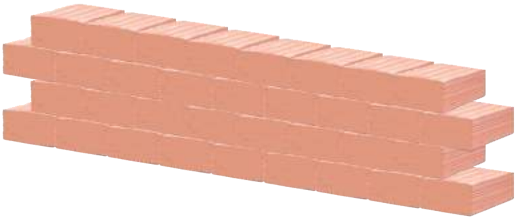
- Homology Theory generalizes the notion of connectivity in Graph Theory to higher dimensions.
- It's defined by a family of groups that capture the number of :
    - Connected components 
    - The number of holes 
    - The number of cavities 

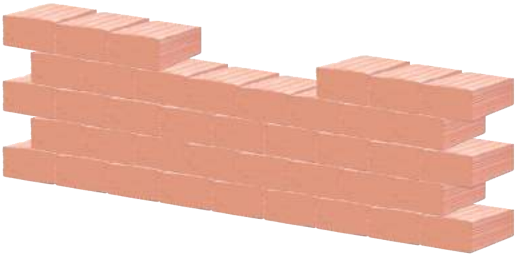
Homology

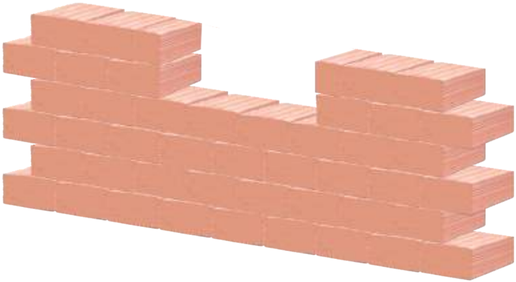
- Homology Theory generalizes the notion of connectivity in Graph Theory to higher dimensions.
- It's defined by a family of groups that capture the number of :
  - Connected components 
  - The number of holes 
  - The number of cavities 
  - The number of such equivalent units features in larger dimension

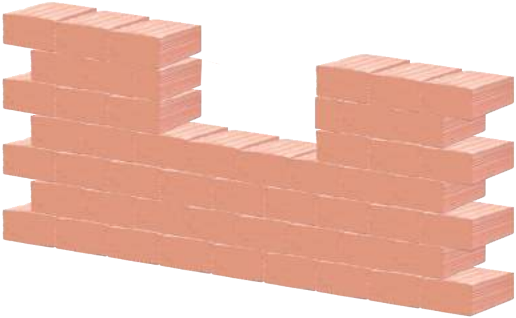


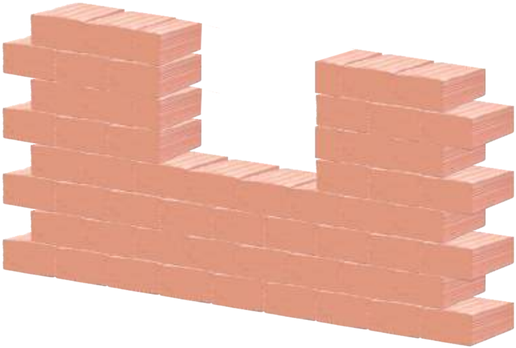


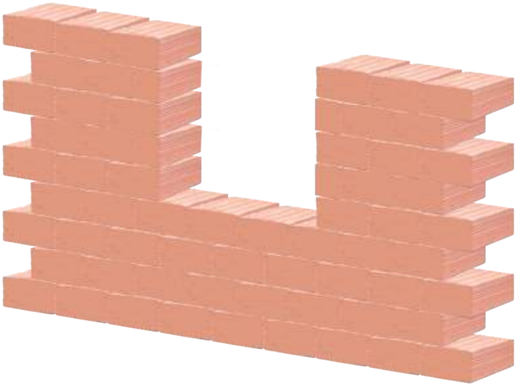


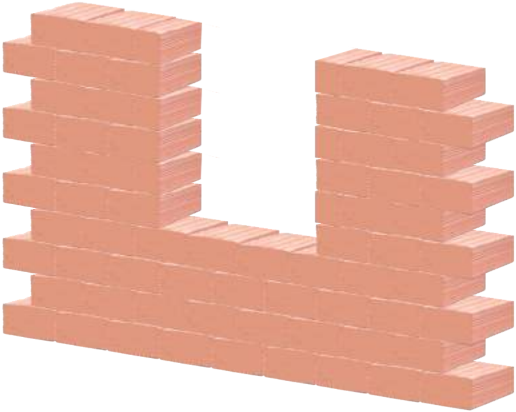


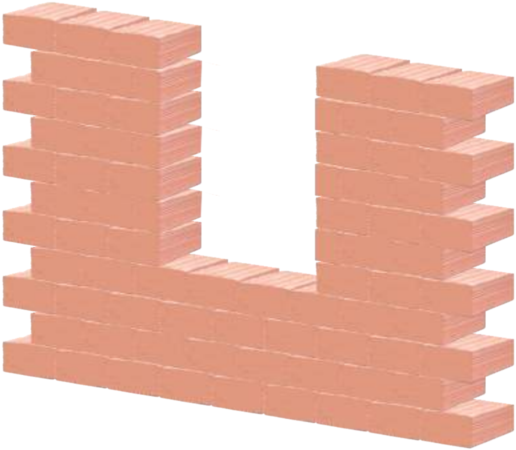


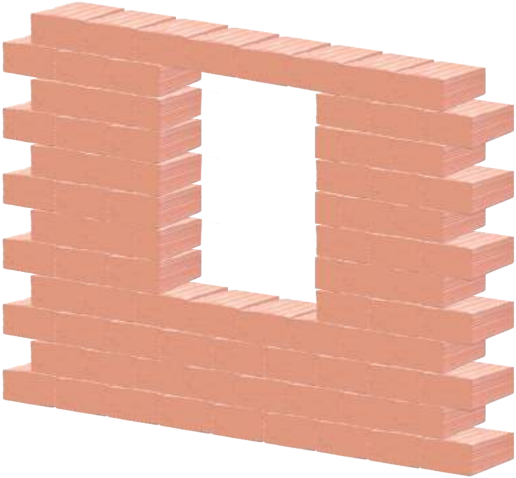


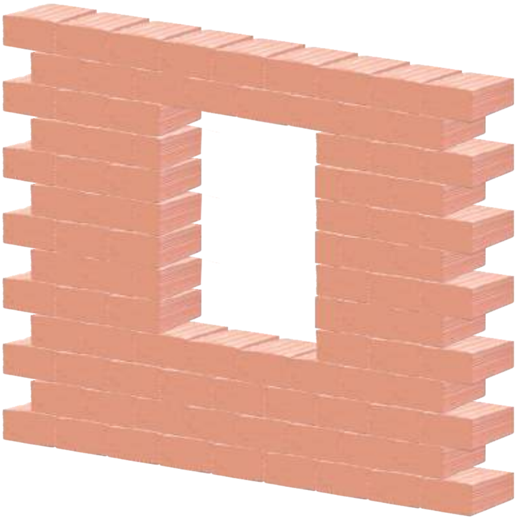


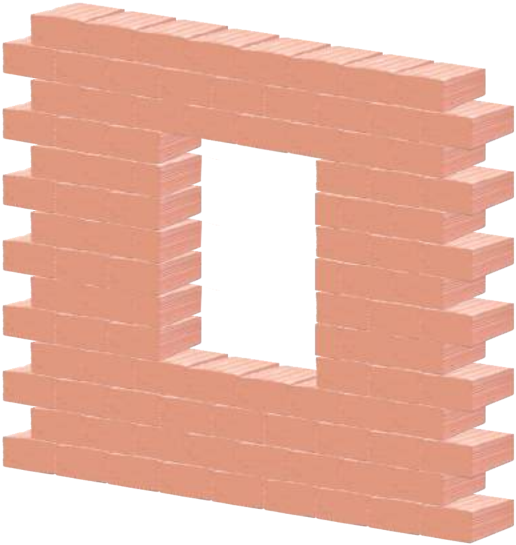


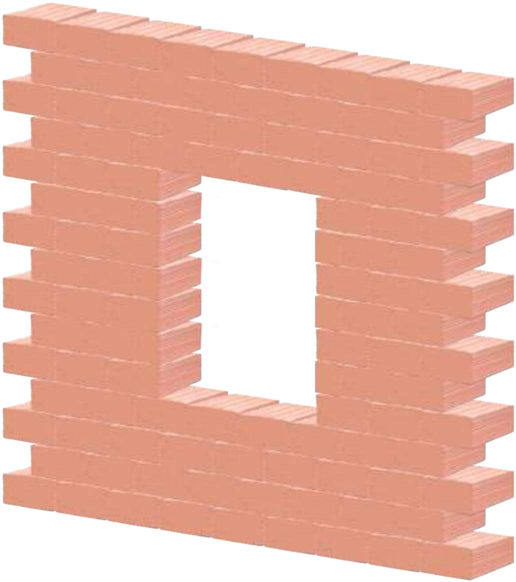


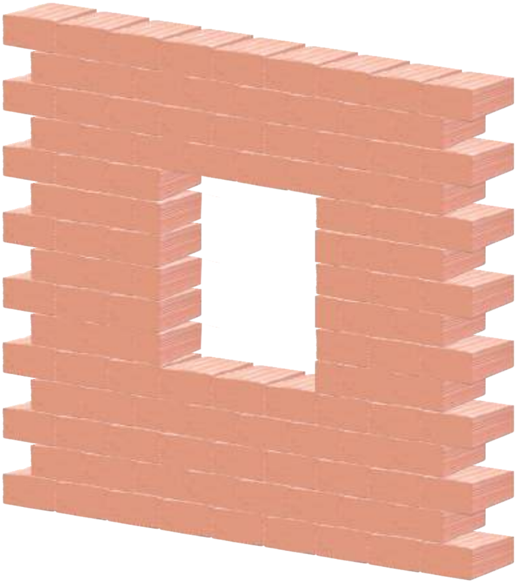


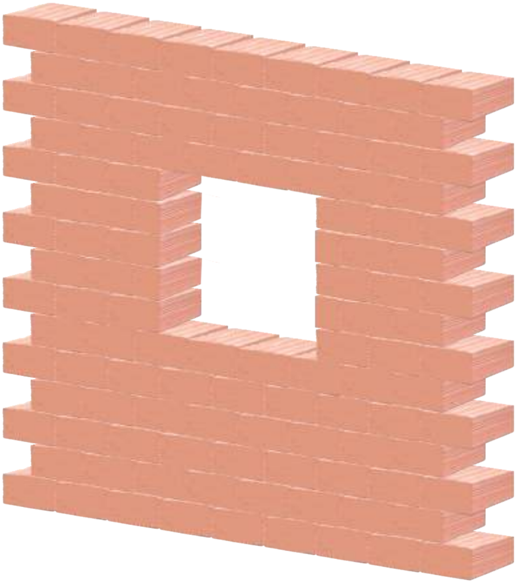


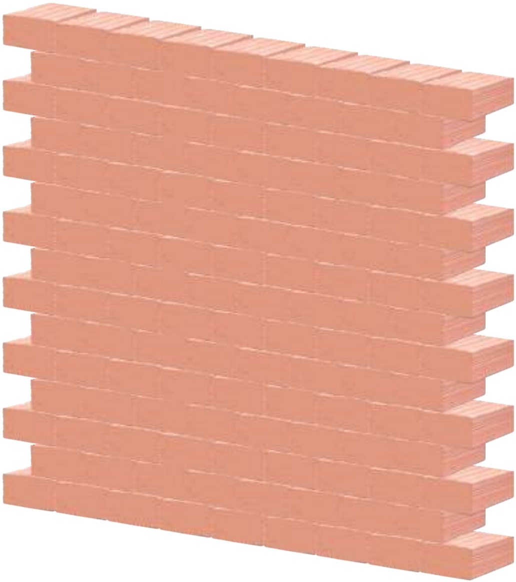


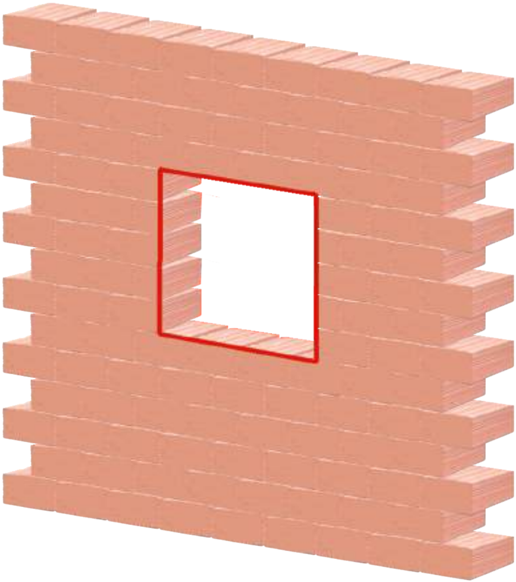


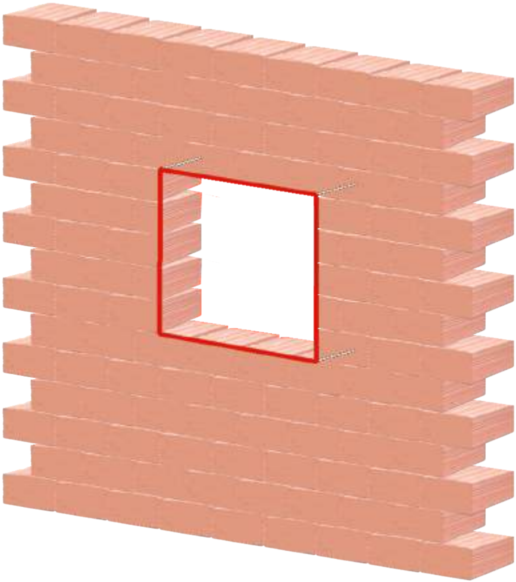


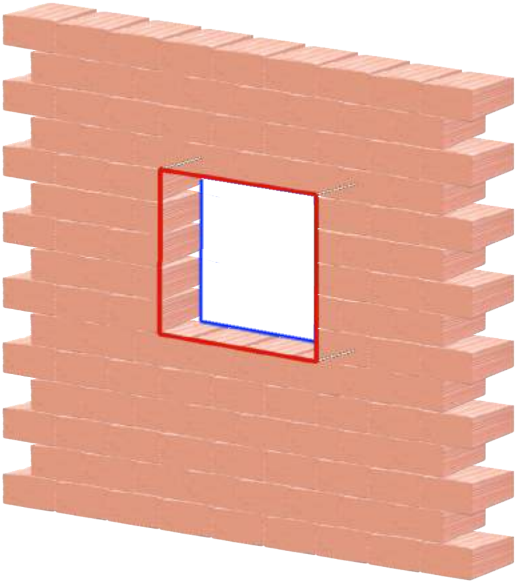


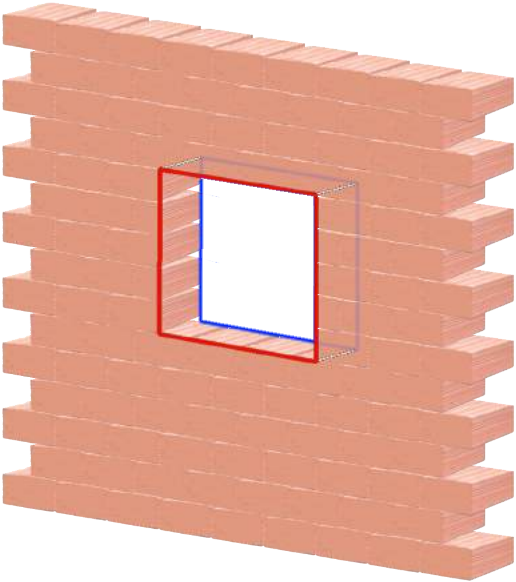


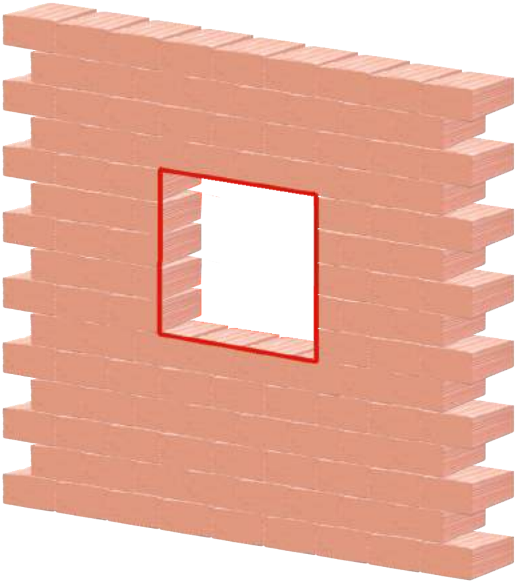


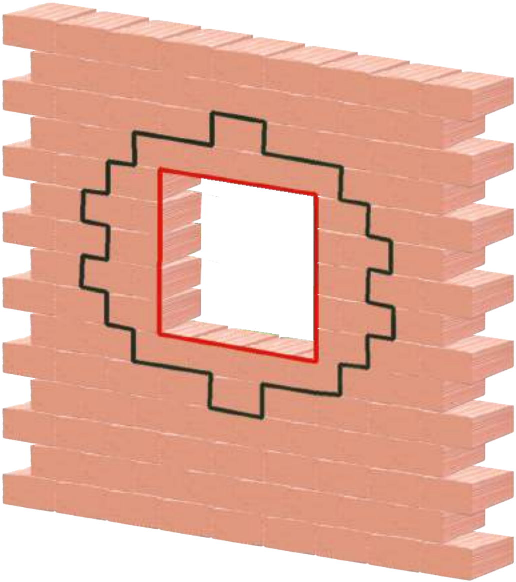


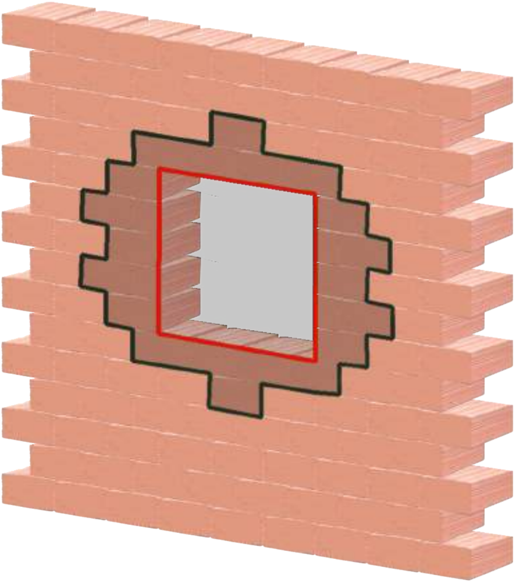




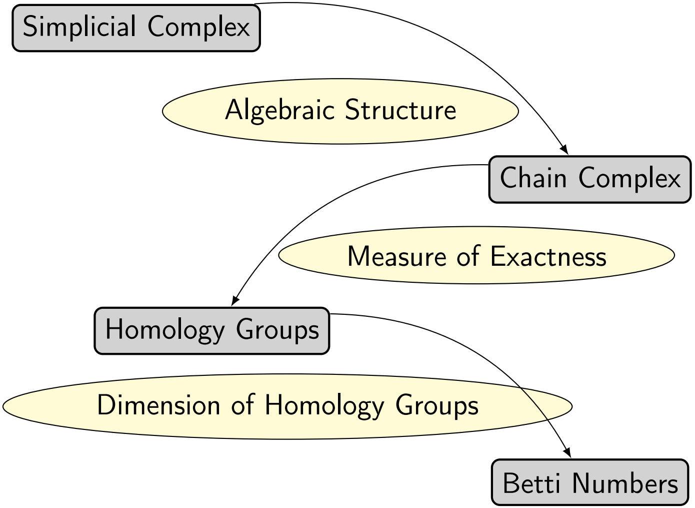








Betti-Numbers and Homology



Sommaire

1 Motivation

2 Introduction

3 Basic Concepts

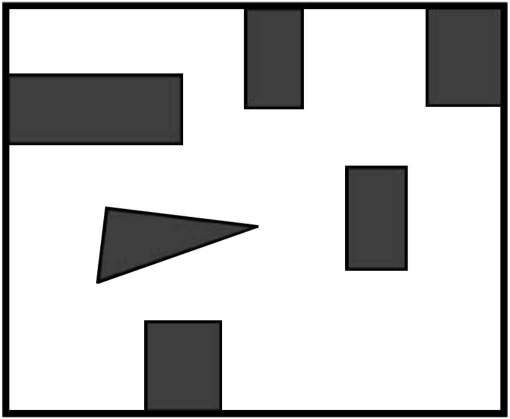
4 Applications

5 Research Focus

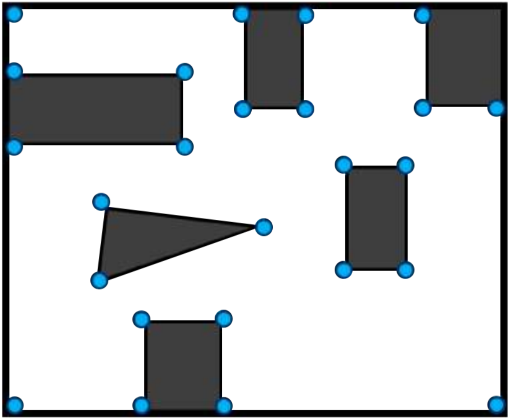
6 Bibliography

Robotics

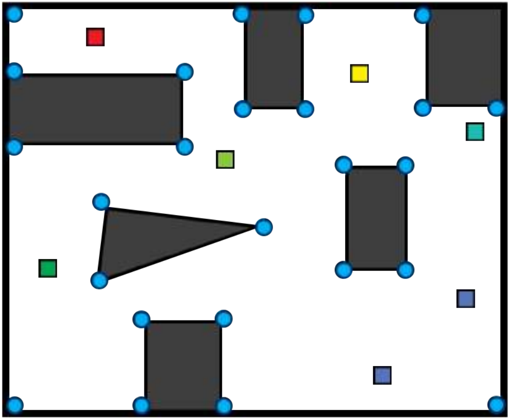
Autonomous Localization



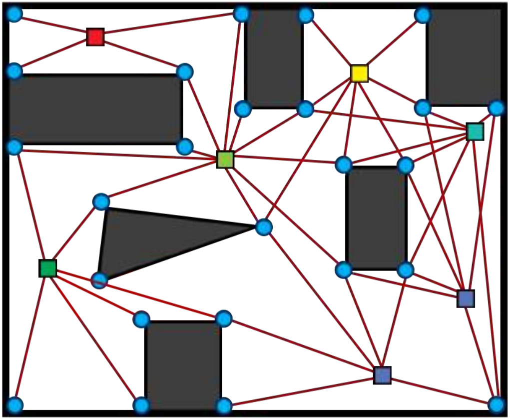
Autonomous Localization



Autonomous Localization



Autonomous Localization



Autonomous Localization

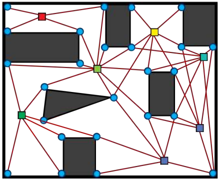


Figure: **Planar domain,D**

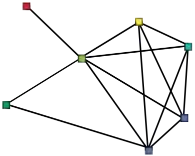


Figure: **Covisibility Network, N**

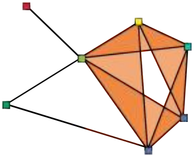


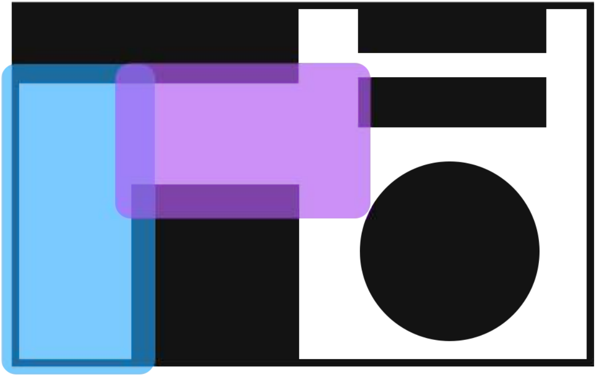
Figure: **Landmark Complex, K**

Neural Coding

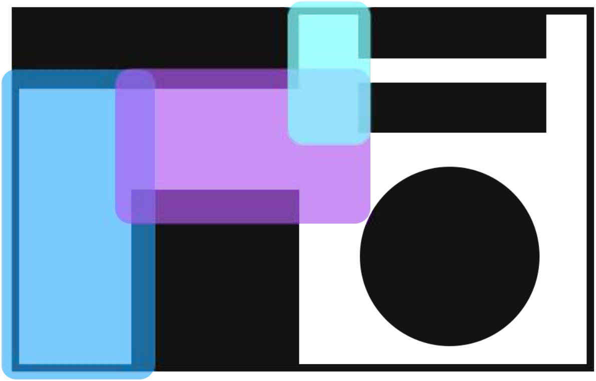
Hippocampal Spatial Map Formation

Place Cells and Place Fields

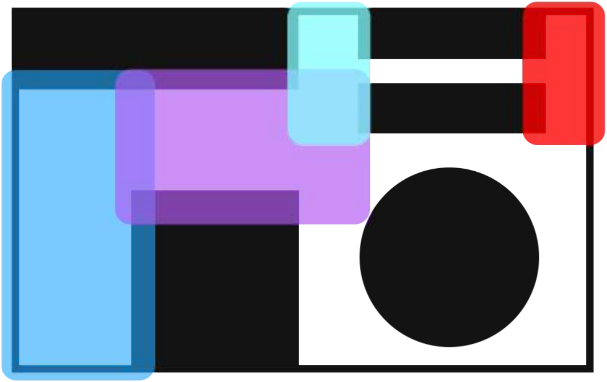
Place Cells and Place Fields



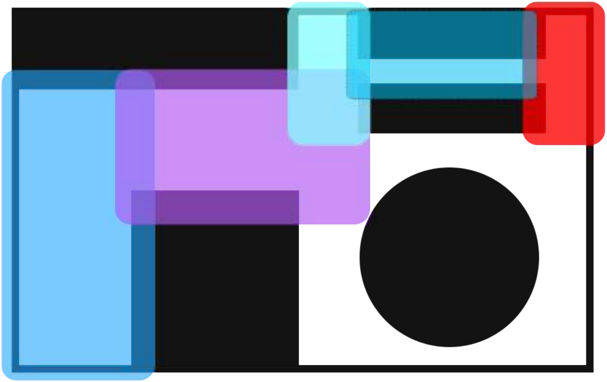
Place Cells and Place Fields



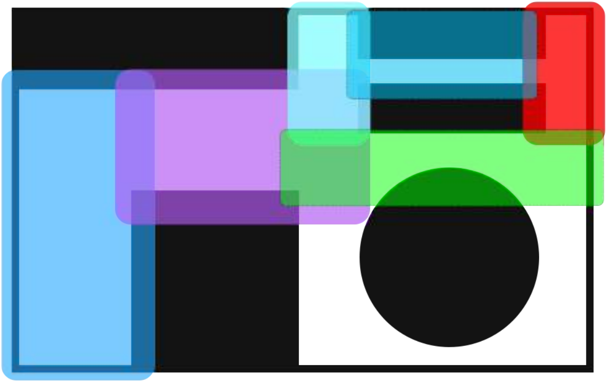
Place Cells and Place Fields



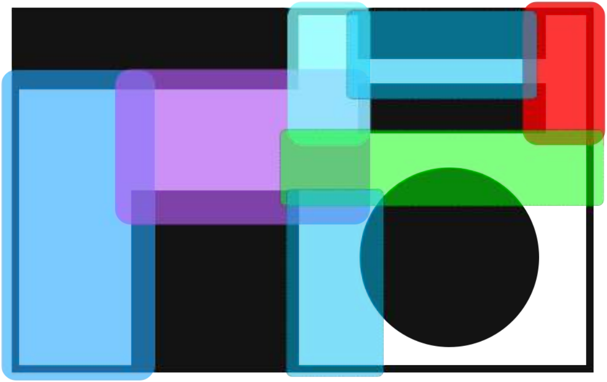
Place Cells and Place Fields



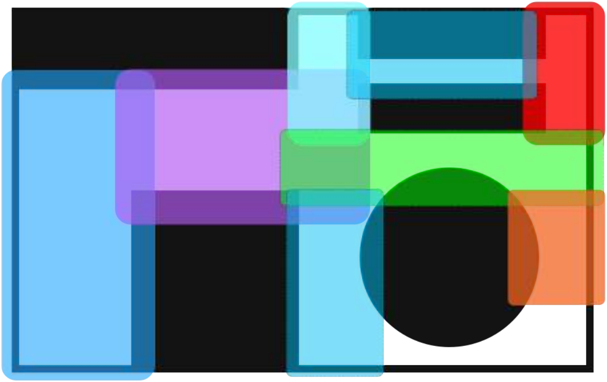
Place Cells and Place Fields



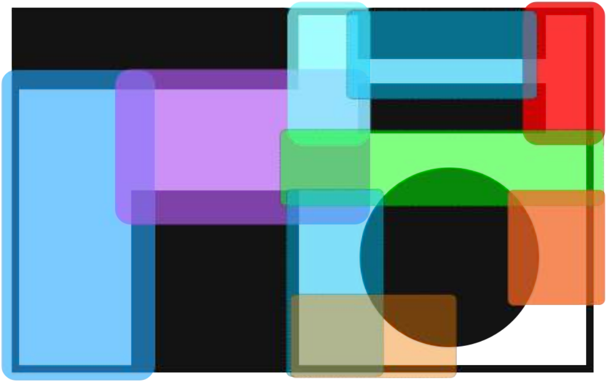
Place Cells and Place Fields



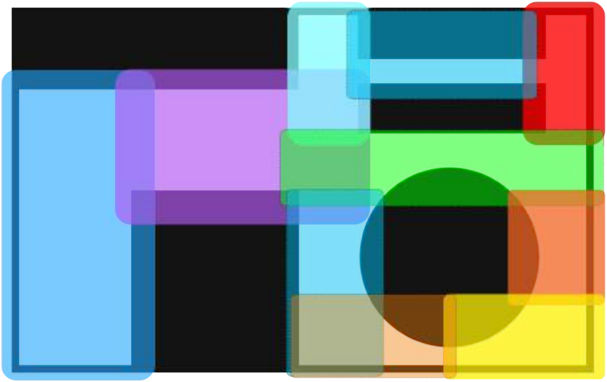
Place Cells and Place Fields



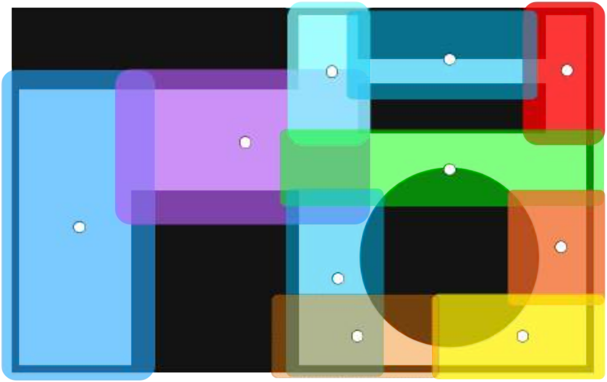
Place Cells and Place Fields



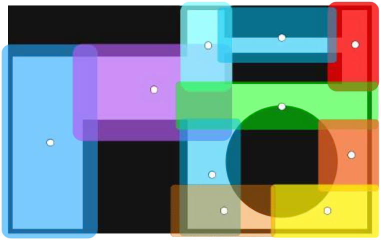
Place Cells and Place Fields

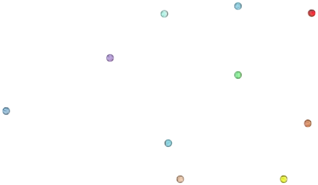


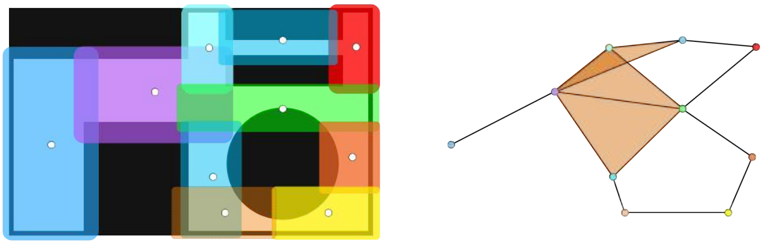
Place Cells and Place Fields



Neural Coding : Hippocampal Spatial Map Formation

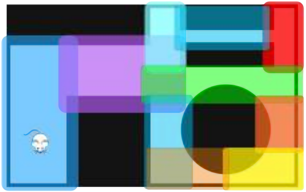




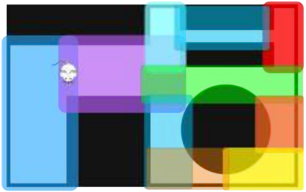


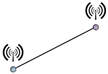
"Pyramidal neurons in rodent hippocampus exhibit a geometric organization due to their role in position coding. Each of these neurons, called place cells, acts as a position sensor, exhibiting a high firing rate when the animal's position lies inside the neuron's place field, its preferred region of the spatial environment" – **Giusti, Clique Topology Reveals Intrinsic Geometric Structure in Neural Correlations.**

Mouse Trajectory

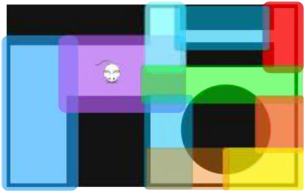


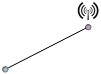
Mouse Trajectory



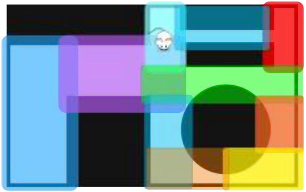


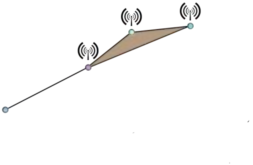
Mouse Trajectory



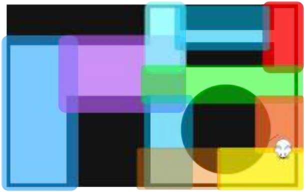


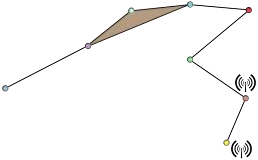
Mouse Trajectory



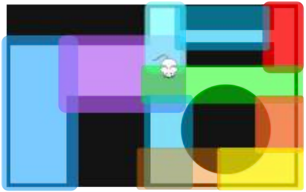


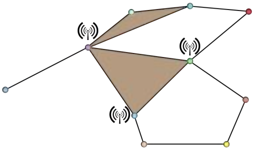
Mouse Trajectory



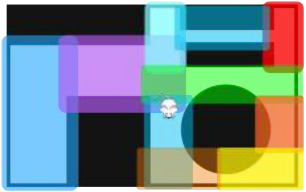


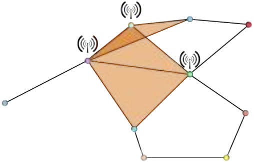
Mouse Trajectory





Mouse Trajectory





Other Application

- Facial Recognition.

Other Application

- Facial Recognition.
- Image processing.

Other Application

- Facial Recognition.
- Image processing.
- Musical Applications.

Other Application

- Facial Recognition.
- Image processing.
- Musical Applications.
- Clinical variation management

Other Application

- Facial Recognition.
- Image processing.
- Musical Applications.
- Clinical variation management
- Financial risk modeling

Sommaire

1 Motivation

2 Introduction

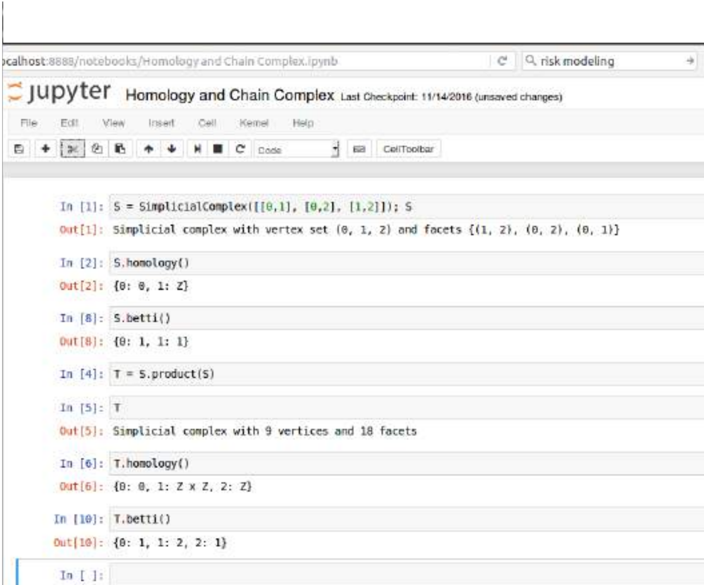
3 Basic Concepts

4 Applications

5 Research Focus

6 Bibliography

Computing Algebraic Topology



Sommaire

1 Motivation

2 Introduction

3 Basic Concepts

4 Applications

5 Research Focus

6 Bibliography

Bibliography

- **A. Hatcher**. *Algebraic Topology*. Cambridge University Press, 2002.
- **Zomorodian AJ**. (2005) *Topology for computing*. Cambridge, UK; New York: Cambridge University Press. xiii, 243 p. p.
- **H. Edelsbrunner, J. Harer**. *Computational Topology*. Duke University. 2009
- **J. Derenick, A. Speranzon, R. Ghrist**. *Homological Sensing for Mobile Robot Localisation*. 2013.
- **R. Ghrist, D. Lipsky, J. Derenick, and A. Speranzon**. *Topological landmark-based navigation and mapping*. University of Pennsylvania, Department of Mathematics, Tech. Rep., 08 2012.
- **Y.Dabaghian, F.Mémoli, L. Frank, G. Carlson**. *A Topological Paradigm for Hippocampal Spatial Map Formation Using Persistent Homology*.Plos Computational Biology.2012 Aug; 8(8): e1002581.

Thanks you for your attention