COSMOS AND ITS FURNITURE I AND II

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Abstract. In this talk I shall continue the study of the geometry of the moduli-space of pairs of points in 3 dimensions. I show that this space, $\underline{\tilde{H}}$, is the base space of a canonical family of associative k -algebras in dimension 4. The study of the corresponding family of derivations leads to a natural way of introducing an action of the gauge Lie algebras of the Standard Model, in $\underline{\tilde{H}}$. The results fit well with the set-up of the Standard Model. It also furnishes a possible mathematical model for a Big Bang-scenario in cosmology. These subjects are all treated within the set-up of [**?**].

Contents

1. Introduction

Mindful of the well known quotation,

> Vos calculs sont corrects, mais votre physique est abominable: (Albert Einstein 1927 à George Lemaître).

I shall start by declaring my cards:

2. Philosophy

If we want to study a natural phenomenon, called **P**, we must in the present scientific situation, describe **P** in some mathematical terms, say as a mathematical object, X, depending upon some parameters, in such a way that the changing aspects of **P** would correspond to altered parameter-values for X. This object would be a *model for* **P** if, moreover, X with any choice of parameter-values, would correspond to some, possibly occurring, aspect of **P**.

2.1. **Moduli spaces.** Two mathematical objects X(1), and X(2), corresponding to the same aspect of **P**, would be called equivalent, and the set, $\mathcal{P}$, of equivalence classes of the objects **P**, would correspond to (a quotient of) the *moduli space*, **M**, of the models, X. The study of the natural phenomena **P**, and its changing aspects, would then be equivalent to the study of the *structure* of $\mathcal{P}$, and therefore to the study of the dynamics of the moduli space **M**.

2.2. **Time.** In particular, the notion of *time* would, in agreement with Aristotle and St. Augustin, see, [**?**] and [**?**], correspond to some *metric* on this space.

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2.3. **Phase Spaces and Dynamical Structure.** It turns out that to obtain a complete theoretical framework for studying the phenomenon **P**, or the model X, together with its *dynamics*, we should introduce the notion of *dynamical structure*, defined for the space, **M**. This is done via the construction of a universal non-commutative *Phase Space*-functor, $Ph(-) : Alg_k \to Alg_k$. It extends to the category of schemes, and its infinite iteration $Ph^\infty(-)$, is outfitted with a universal *Dirac derivation*, $\delta \in Der_k(Ph^\infty(-), Ph^\infty(-))$. A dynamical structure defined on an associative k-algebra $A \in Alg_k$ is now a δ-stable ideal $\sigma \subset Ph^\infty(A)$, and its quotient $A(\sigma) := Ph^\infty(A)/(\sigma)$, with its Dirac derivation. The structure we are interested in is the *space* $\mathbf{U} := Ph^\infty(\mathbf{M})/\sigma$, corresponding to an open affine covering by algebras of the type, $A(\sigma)$, see [**?**], [**?**], see also [**?**].

2.4. **Gauge Lie Algebras.** But now we observe that there may be an action of a Lie algebra $\mathfrak{g}$, on **U**, such that the dynamics of $\mathcal{P}$, really, corresponds to that of the quotient $\mathbf{U}/\mathfrak{g}$.

2.5. **Noncommutative Algebraic Geometry.** To any *open* subset O, of **U**, there would be associated a, not necessarily commutative, affine k-algebra, $A(\sigma) := O_{\mathbf{U}}(O)$, with an action of the Lie algebra $\mathfrak{g}$, such that the non-commutative quotient $O/\mathfrak{g}$, represented by the system of simple representations of the algebra, $A(\sigma)(\mathfrak{g})$, would contain all the available information about the structure of O.

2.6. **Quantum Theory.** An element of this algebra would be called an *observable*, and wishing to measure the *value* of an observable, leads to the study of the eigenvectors, and their eigenvalues of the representations of this algebra.

3. The Mathematical Framework

With this philosophy in mind, and stimulated by the results in deformation theory, obtained in [**?**], and [**?**], we embarked, in a series of papers, see [**?**], [**?**], [**?**], on the study of moduli spaces of representations (modules) of associative algebras, in general, and on their quotients, modulo Lie algebra actions. Here is where *invariant theory* and *non-commutative algebraic geometry* enters the play. The Dirac derivation translates into a vector field on these moduli spaces, and give us the equations of motions that we need.

In [**?**], [**?**], see also [**?**], we introduced a *toy model*, used to illustrate the general theory, and to connect to present days physics. It was shown to generalize both general relativity and quantum field theory. In particular, the definition of time fits well with the notion of time in both quantum Yang-Mills theory and in General Relativity, where it made the space of velocities compact.

In this talk, this toy model has become the main figure, furnishing a (nice, but maybe not too realistic) mathematical model for a Big Bang scenario for the universe. The *Cosmos and its Furniture* of the title refer to this model, to its *geometry*, defined by a metric, it's dynamical structure defined by a Dirac derivation, and its *material content*, called it's *furniture*.

The basic idea is that if one choose to take the Big Bang idea seriously, one would, probably, have to accept the presence of a *singularity* at the *start of the universe*. But then one might guess upon a mathematical model for this singularity, and use deformation theory, and the machinery described above to unravel the universe that we see. This is what we do, in chapter 3. We start with the basic singularity in dimension 3. It is composed of a single point, and a 3-dimensional tangent space, with affine algebra given by,

$$U = k[x_1, x_2, x_3]/(x_1, x_2, x_3)^2.$$

The base space of the miniversal deformation of this algebra, as an associative algebra, turns out to contain the above toy-model, and a lot of structure, with strange and maybe interesting interpretations in physics, in particular it seems that it contain a mathematically reasonable basis for the Standard model.

4. Linear Description

The talk will be organized to contain either the Chapters, (1), (2), (4) or (5), or (1), (2), (3). depending upon interest and time.

Chapter 1. is an introduction to the general method proposed in [**?**], with a short reminder of the most important technical tools.

Chapter 2. contains a reworking of the toy model, in particular including the notion of *furniture*, and the related Schrødinger equation, generalizing the Heat equation and the Navier-Stokes.

Chapter 3. contains a short introduction to an algebraic version of Entropy, and related to this, compute the moduli-space of finite dimensional representations of the infinite phase space of a polynomial algebra.

Chapter 4. contains the Cosmological Model described above.

Chapter 5. is then concerned with the deformation theory of the 4 -dimensional associative algebras U, and with the structure of its miniversal base space. The fact that this space is containing the toy model, provides new and unsuspected structures for this model. The main result is the existence, produced by the deformation theory of associative algebras, of a canonical gauge Lia-algebra bundle, containing the gauge algebras of the Standard Model and a strange *super symmetry* relating the spaces of, our versions of, *bosonic* and *fermionic* fields.

References

[1] St. Augustin *Les Confessions de Saint Augustin* par Paul Janet. Charpentier, Libraire-Éditeur, Paris (1861).

[2] Harald Bjar and O. A. Laudal (1990) *Deformations of Lie algebras, and Lie algebras of Deformations* Comp. Math.75:69-111,1990. .

[3] E.Eriksen, O.A.Laudal, A.Siqveland Noncommutative Deformation Theory Book in preparation.

[4] Brian Greene The Fabric of the Cosmos, Allen Lane 2004

[5] Kitty Ferguson (2010) *Pythagoras. His lives and the legacy of a rational universe* Icon Books Ltd.

[6] O. Gravir Imenes (2011) *The concept of charge in a model based on non-commutative algebraic geometry.* Dissertation presented for the degree of Philosophiae Doctor. Faculty of Mathematics and Natural Sciences, University of Oslo, September 2011.

[7] Stephen Hawking (1988) *A Brief History of Time.* Bantam Books, Transworld Publishers, London. (1988)

[8] O. A. Laudal (1979) *Formal moduli of algebraic structures* Lecture Notes in Math.754, Springer Verlag, (1979).

[9] O. A. Laudal (1986) *Matric Massey products and formal moduli I* in (Roos, J.E.ed.) Algebra, Algebraic Topology and their interactions Lecture Notes in Mathematics, Springer Verlag, vol 1183, (1986) pp. 218–240.

[10] O. A. Laudal (2002) *Noncommutative deformations of modules* Special Issue in Honor of Jan-Erik Roos, Homology, Homotopy, and Applications, Ed. Hvedri Inassaridze. International Press, (2002).See also: Homology, Homotopy, Appl. 4(2002), pp. 357-396,

[11] O. A. Laudal (2000) *Noncommutative Algebraic Geometry* Max-Planck-Institut für Mathematik, Bonn, 2000 (115).

[12] O. A. Laudal (2001) *Noncommutative algebraic geometry* Proceedings of the International Conference in honor of Prof. Jos Luis Vicente Cordoba, Sevilla 2001. Revista Matematica Iberoamericana.19 (2003), 1-72.

[13] O. A. Laudal (2003) *The structure of* $Simp_n(A)$ (Preprint, Institut Mittag-Leffler, 2003-04.) Proceedings of NATO Advanced Research Workshop, Computational Commutative and Non-Commutative Algebraic Geometry. Chisinau, Moldova, June 2004.

[14] O. A. Laudal (2005) *Time-space and Space-times* Conference on Noncommutative Geometry and Representatioon Theory in Mathematical Physics. Karlstad, 5-10 July 2004. Ed. Jürgen Fuchs, et al. American Mathematical Society, Contemporary Mathematics, Vol. 391, 2005. ISSN: 0271-4132.

[15] O. A. Laudal (2007) *Phase Spaces and Deformation Theory* Preprint, Institut Mittag-Leffler, 2006-07. See also the part of the paper published in: Acta Applicanda Mathematicae, 25 January 2008.

[16] O. A. Laudal (2007) *Geometry of Time Spaces* World Scientific, (2011).

[17] O. A. Laudal (2007) *Cosmos and its Furniture* Proceedings Rabat/Lahore 2013..

[18] O. A. Laudal and G. Pfister (1988) *Local moduli and singularities* Lecture Notes in Mathematics, Springer Verlag, vol. 1310, (1988)

[19] J.Madore, S. Schraml, P. Schupp, J. Wess (2000) *Gauge Theory on Noncommutative Spaces* arXiv:hep-th/0001203v1 28 Jan 2000.

[20] F.Mandl and G. Shaw (1984) *Quantum field theory* A Wiley-Interscience publication. John Wiley and Sons Ltd. (1984)

[21] R. Penrose *What came before the Big Bang? Cycles of Time*Vintage Books (2011)

[22] C. Procesi (1967) *Non-commutative affine rings* Atti Accad. Naz. Lincei Rend.Cl. Sci. Fis. Mat. Natur. (8)(1967), 239-255

[23] C. Procesi (1973): Rings with polynomial identities. Marcel Dekker, Inc. New York, (1973).

[24] R.K.Sachs and H.Wu (1977) *General Relativity for Mathematicians* Springer Verlag, (1977)

[25] T. Schücker (2002) *Forces from Connes' geometry* arXiv:hep-th/0111236v2, 7 June 2002.

[26] N. Seiberg and E.Witten (1999) *String Theory and Noncommutative Geometry* JHEP 9909,032(1999),hep-th/9908142.

[27] S. Weinberg (1995) *The Quantum Theory of Fields. Vol I, II, III* Cambridge University Press, (1995).

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